

Model Question Paper

Reg No:

Name:

RAJAGIRI SCHOOL OF ENGINEERING & TECHNOLOGY

(AUTONOMOUS)

SECOND SEMESTER B.TECH DEGREE EXAMINATION, AUGUST/SEPTEMBER 2021

Course Code:100908 /MA 200A

**Course name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND
TRANSFORMS**

Max. Marks: 100

Duration: 3 Hours

PART A

(Answer all questions. Each question carries 3 marks)

1. Is the vector \mathbf{r} where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ conservative. Justify your answer.
2. State Greens theorem including all the required hypotheses.
3. What is the outward flux of $(x, y, z) = xi + yj + zk$ across any unit cube.
4. What is the relationship between Green's theorem and Stokes theorem?
5. Solve $y'' + 4y' + 2.5y = 0$.
6. Does the function $y = C_1 \cos x + C_2 \sin x$ form a solution of $y'' + y = 0$?. Is it the general solution? Justify your answer.
7. Find the Laplace transform of $e^{-t} \sinh 4t$.
8. Find the Laplace inverse transform of $F(s) = \frac{2s}{s^2+2}$.
9. Find the Fourier transform of $f(x) = e^{-x}$.
10. State the convolution theorem for Fourier transform.

PART B

(Answer one full question from each module. Each full question carries 14 marks)

MODULE 1

- 11 a) Prove that the force field $\mathbf{F} = e^y\mathbf{i} + xe^y\mathbf{j}$ is conservative in the entire xy-Plane.
b) Use Greens theorem to find the area enclosed by the ellipse
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$
- 12 a) Find the divergence of the vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

b) Find the work done by the force field $(x, y, z) = xyi + yzj + xzk$ along C where C is the curve $r(t) = ti + t^2j + t^3k$.

MODULE II

13 a) Use divergence theorem to find the outward flux of the vector field $F = 2xi + 3yj + z^3k$ across the unit cube bounded by $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$.

b) Find the circulation of $F = (x - z)i + (y - x)j + (z - xy)k$ using Stokes theorem around the triangle with vertices $A(1,0,0), B(0,2,0)$ and $C(0,0,1)$.

14 a) Use divergence theorem to find the volume of the cylindrical solid bounded by $x^2 + 4x + y^2 = 7, z = -1, z = 4$, given the vector field $F = xi + yj + zk$ across surface of the cylinder.

b) Use Stokes theorem to evaluate $\int_C F \cdot dr$ where $F = x^2i + 3xj - y^3k$ where C is the circle $x^2 + y^2 = 1$ in the xy- plane with counterclockwise orientation looking down the positive z-axis.

MODULE III

15 a) Solve $y'' + 4y' + 4y = x^2 + e^{-x} \cos x$.

b) Solve $y''' - 3y'' + 3y' - y = e^x - x - 1$.

16 a) Solve $y''' + 3y'' + 3y' + y = 30e^{-x}$ given $y(0) = 3, y'(0) = -3, y''(0) = -47$.

b) Using method of variation of parameters, solve $y'' + y = \sec x$.

MODULE IV

17 a) Find the inverse Laplace form of $F(s) = \frac{3s+7}{s^2+2s+9}$.

b) Solve the differential equation $y'' + 16y = 4\delta(t - 3\pi); y(0) = 2, y'(0) = 0$

using Laplace transform.

18 a) Solve $y'' + 3y' + 2y = f(t)$ where $f(t) = 1$ for $0 < t < 1$ and $f(t) = 1$ for $t > 1$ using Laplace transform.

b) Apply convolution theorem to find the Laplace inverse transform of

$$F(s) = \frac{s^2}{s^2 + 6s + 9}.$$

MODULE V

19 a) Find the Fourier cosine integral representation for $f(x) = e^{-kx}$ for $x > 0$ and $k > 0$.

b) Does the Fourier sine transform $(x) = x^{-1} \sin x$ for $0 < x < \infty$ exist? Justify your answer.

20 a) Find the Fourier transform of $f(x) = |x|$ for $|x| < 1$ and $f(x) = 0$ otherwise.

b) Find the Fourier cosine transform of $(x) = e^{-ax}$ for $a > 0$.
